XXVII Reunión de la Asociación Latinoamericana de Producción Animal (ALPA)
XLVII Reunión de la Asociación Mexicana de Producción Animal y Seguridad Alimentaria, A. C. (AMPA)
Palacio de Convenciones. Zacatecas, Zac., México. Abril 26-28, 2023.
www.doi.org/10.53588/alpa.310553

The shape of curve lactation of crossbred dairy sheep affects the fitting of empirical and mechanistic models

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Abstract. The ability of mathematical models to represent the lactation process varies according with their mathematical structure and database characteristics. The aim of the current study was evaluated the goodness of fit of empirical and mechanistic models applied to dairy sheep lactation curves with different shapes. A total of 4,494 weekly test day records were analyzed. All lactations were individually fitted using two empirical (Wood and Wilmink) and two mechanistic (Dijkstra, and Pollott) models. The Dijkstra model showed the best performance to typical curves and Wood model to atypical curves (without peak lactation). Therefore, the selection of the mathematical model to fit sheep lactation curves must consider the specific patter of milk production.

Keywords: sheep, lactation, models, dairying

La forma de la curva de lactancia de las ovejas lecheras cruzadas afecta el ajuste de los modelos empíricos y mecanísticos.

Resumen. La capacidad de los modelos matemáticos para representar el proceso de lactancia varía según su estructura matemática y las características de la base de datos. El objetivo del presente estudio fue evaluar la bondad de ajuste de modelos empíricos y mecanísticos aplicados a curvas de lactación de ovejas lecheras con diferentes formas. Se analizaron un total de 4.494 registros de días de prueba semanales. Todas las lactancias se ajustaron individualmente utilizando dos modelos empíricos (Wood y Wilmink) y dos mecanísticos (Dijkstra y Pollott). El modelo de Dijkstra mostró el mejor desempeño a las curvas típicas y el modelo de Wood a las curvas atípicas (sin pico de lactancia). Por lo tanto, la selección del modelo matemático para ajustar las curvas de lactancia de las ovejas debe considerar el patrón específico de producción de leche.

Palabras clave: ovejas, lactancia, modelos, lechería

A forma da curva de lactação de ovelhas leiteiras mestiças afeta o ajuste de modelos empíricos e mecanísticos.

Resumo. A capacidade dos modelos matemáticos em representar o processo de lactação varia de acordo com sua estrutura matemática e características do banco de dados. O objetivo do presente estudo foi avaliar a qualidade do ajuste de modelos empíricos e mecanísticos aplicados a curvas de lactação de ovelhas leiteiras com diferentes formas. Um total de 4.494 registros de dia de teste semanais foram analisados. Todas as lactações foram ajustadas individualmente usando dois modelos empíricos (Wood e Wilmink) e dois modelos mecanísticos (Dijkstra e Pollott). O modelo de Dijkstra apresentou melhor desempenho para curvas típicas e o modelo de Wood para curvas atípicas (sem pico de lactação).

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Portanto, a seleção do modelo matemático para ajustar as curvas de lactação de ovelhas deve considerar o padrão específico de produção de leite.

Palavras-chave: ovinos, lactação, modelos, leite.

Introduction

The analysis of dairy sheep production using mathematical models allows identifying the lactation curve's characteristics to develop and implement strategies (Angeles-Hernandez *et al.*, 2018). However, the ability to represent the lactation process varies among mathematical models mainly due to its mathematical structure. In this sense, each model can have a certain number of parameters, which can have a biological interpretation or represent the geometry of the lactation (Morant & Gnanasakthy, 1989). The best model is the that provides adequate estimates of total milk yield (TMY), maximum milk production, and the time at which it occurs. Also, a good lactation curve model must describe the ability of the animal to maintain milk production over time during the descending phase regardless of the lactation curve's shape (Rodríguez et al., 2005).

Lactation curve models can be classified as empirical and mechanistic. Empirical models are used to find the mathematical equation that best fits experimentally (Giraldo *et al.*, 2002). While mechanistic models are based on understanding the components of the system with a biological interpretation of its parameters (Dijkstra *et al.*, 1997). Some of these models are Wood (1967) and Wilmink (1987) as empirical models and Dijskstra (1997) and Pollot (2000) as mechanistic models. The lactation model research has focused mainly on dairy cows. Consequently, there are few studies on sheep. Therefore, the current study aimed to evaluate the effect of the lactation curve on the goodness of fit to Wood, Wilmink, Dijkstra, and Pollot models applied to dairy sheep lactation in Mexico.

Materials y Methods

This research was performed in a sheep farm dedicated to milk production raised in Queretaro, Mexico (20°31´N, 100°24´W) with mean annual temperature of 17.3°C and average annual rainfall of 485 mm. A total of 4,494 weekly test day records from 156 lactations were analyzed in the current study. We analyzed lactation curves of dairy crossbred ewes of the following breeds: East Friesian, Pelibuey, Suffolk and Black Belly. Ewes were milked mechanically, and milk yields were recorded once per week. The actual total milk yield was calculated using the centering date method (Sargent, 1968), and all lactations were individually fitted using the following models:

Wood model or incomplete gamma model (WD; Wood, 1967):

$$Y = at^{b} e^{-ct.} \qquad (2)$$

Where Y is the milk yield at time t; *a* is the parameter that represents the milk yield at the beginning of lactation; and *b* and *c* are the parameters of inclining and declining slopes of the lactation curve before and after the peak production.

Wilmink model or wilmink's exponential model (WL; Wilmink, 1987):

$$Y=a + be^{kt} + ct \qquad (3)$$

Where Y is the milk yield at time t; *a* parameter is associated with the milk production level; *b* and *c* parameters represent the increasing and decreasing of milk production around the peak lactation; k parameter assumes a fixed value derived from a preliminary analysis and is associated with the time at peak yield, for the current study, the values of k parameter was 0.033.

Dijkstra model (DJ; Dijkstra et al., 1997)

$$Y=a*\exp((b/c)*(1-\exp(-c*t))-(d*t) \qquad (4)$$

Where Y is the milk yield at time t; *a* is the theoretical initial milk production; *b* is the specific rate of secretory cell proliferation at parturition; *c* is the decay parameter; and *d* is the specific rate of secretory cell death.

Pollott model (PO; Pollott, 2000)

$$Y=(a/(1+((1-0.9999999)/0.9999999)*\exp(-b*(t-150))))*(2-\exp(c*t)) \qquad (5)$$

Where Y is the milk yield at time t; *a* is the maximum milk secretion potential of the lactation; *b* is the relative proliferation rate of secretory cell number during early lactation; *c* is the relative decline rate in cell number. For the current study, a reduced version of the Pollott model was used to test models with a similar number of parameters.

The different shapes of lactation curves were tested based on the parameters proprieties of each model. For Wood model when parameters b and c correspond, respectively, to positive and negative values (b<0 and c>0), a standard (typical) curve is found. In the Wilmink model, when b and c are negative (b or c >0) there is a typical curve (Macciotta et al., 2005). For Dijkstra and Pollott models, atypical curves were identified by visual inspection after to graphic mathematical functions.

The parameters of models were estimated using the "minpack.lm" package in the R software (Elzhov et al., 2022). The lactation curve characteristics, TMY, peak yield (PY), and time at PY (TPY) were calculated for each lactation. The ability of mathematical models to fit typical and atypical lactation curves was evaluated using the mean square of prediction error (MSPE), Root of MSPE (RMSPE), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and the coefficient of correlation (r) between the actual and estimated TMY.

Results

The goodness of fit of empirical and mechanistic models according to the shape of lactation curves is presented in Table 1. The best TMY estimates of typical curves were observed in the WL model; the remaining models overestimated TMY by 4.5 to 9.5 l (Table 1). Regarding atypical curves, all models overestimated TMY; with the best estimates showed by the PO model according to the r values (0.98***) (Fig. 2).

Table 1. Goodness of fit evaluation of dairy sheep lactation fitted using mechanistic and empirical models.

	Typical					Atypical				
	Actual	Wood	Wilmink	Dijkstra	Pollott	Actual	Wood	Wilmink	Dijkstra	Pollott
TMY (l)	98.3	104.5	95.9	107.1	102.8	90.3	110.7	104.3	109.0	105.8
TPY (d)	46.33	37.38	37.93	44.97	34.44	23.12	1.9	1.00	7.76	4.04
PY (l)	0.92	0.72	0.67	0.79	0.70	0.92	6.63	1.13	5.23	0.66
MSPE		0.014	0.016	0.013	0.017		0.010	0.012	0.012	0.019
RMSPE (l)		0.11	0.12	0.11	0.12		0.10	0.10	0.10	0.13
r		0.90**	0.90***	0.99**	0.90***		0.82**	0.96***	0.61***	0.98***

*TMY: Total milk yield; TPY: Time of peak yield; PY: Peak yield; MSPE: Mean square of prediction error; RMSPE: Root of MSPE; r: coefficient of correlation.

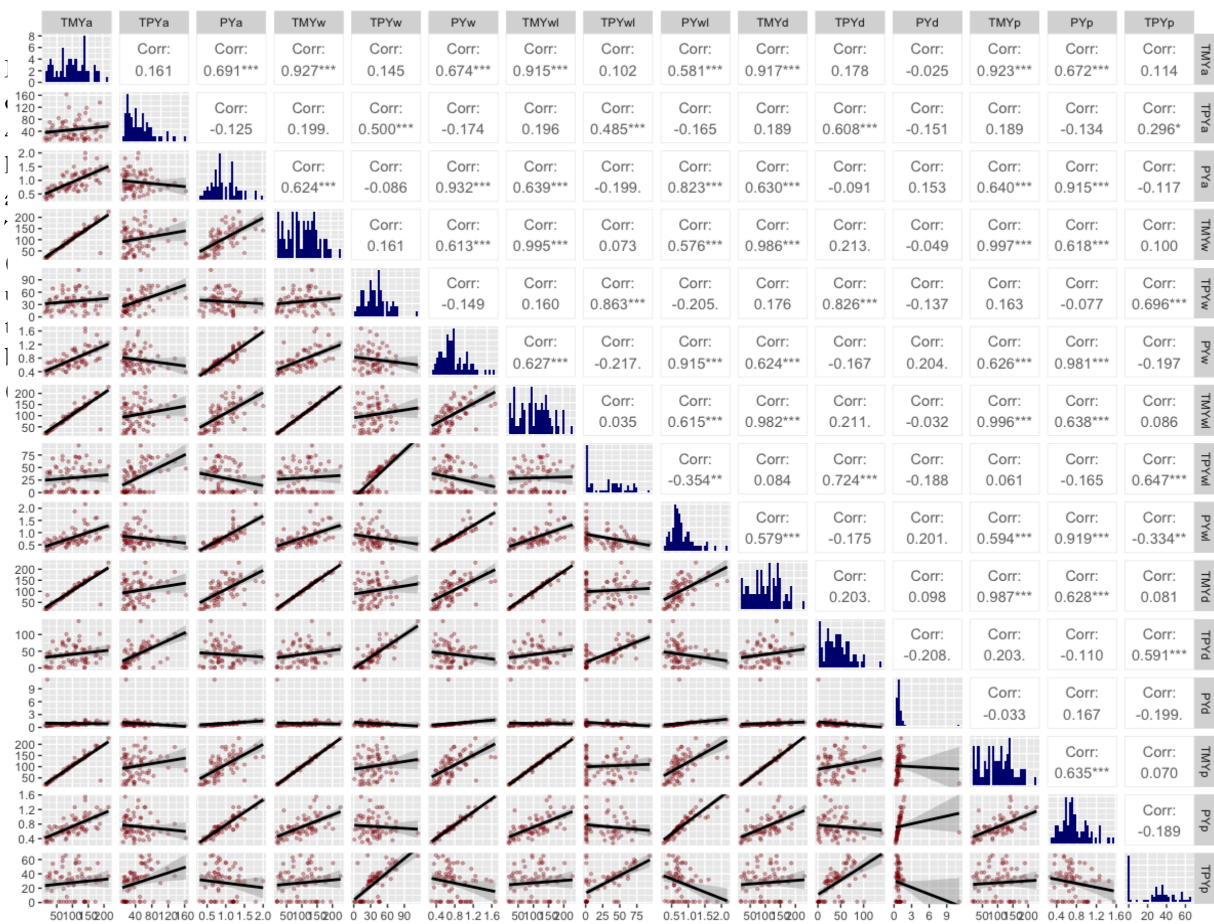

Figure 1. Correlogram of actual and estimated characteristics of typical lactation curves.

Most of the models overestimated the PY of atypical curves, except the PO model, which also had the highest values of r (0.81***) (Fig. 2). According to the MSPE and RMSPE, the four models showed adequate accuracy in predicting the milk yield of dairy sheep. The RMSPE allows calculating the error of estimate in the same unit as the actual value. In this sense, the estimated error range of milk yield for the four models was 0.11 to 0.12 l for typical curves and 0.10 to 0.13 l for atypical curves (Table 1).

Based on the criteria of goodness of fit, the DJ model had the best-fit performance in typical curves, with the low values of AIC (-37.17) and BIC (-31.1), followed by the WD model (Fig. 3). For atypical curves, the best goodness of fit was found for the WD model. Meanwhile, the PO model showed difficulties fitting lactation with an atypical shape based on the values of AIC and BIC (Fig. 3

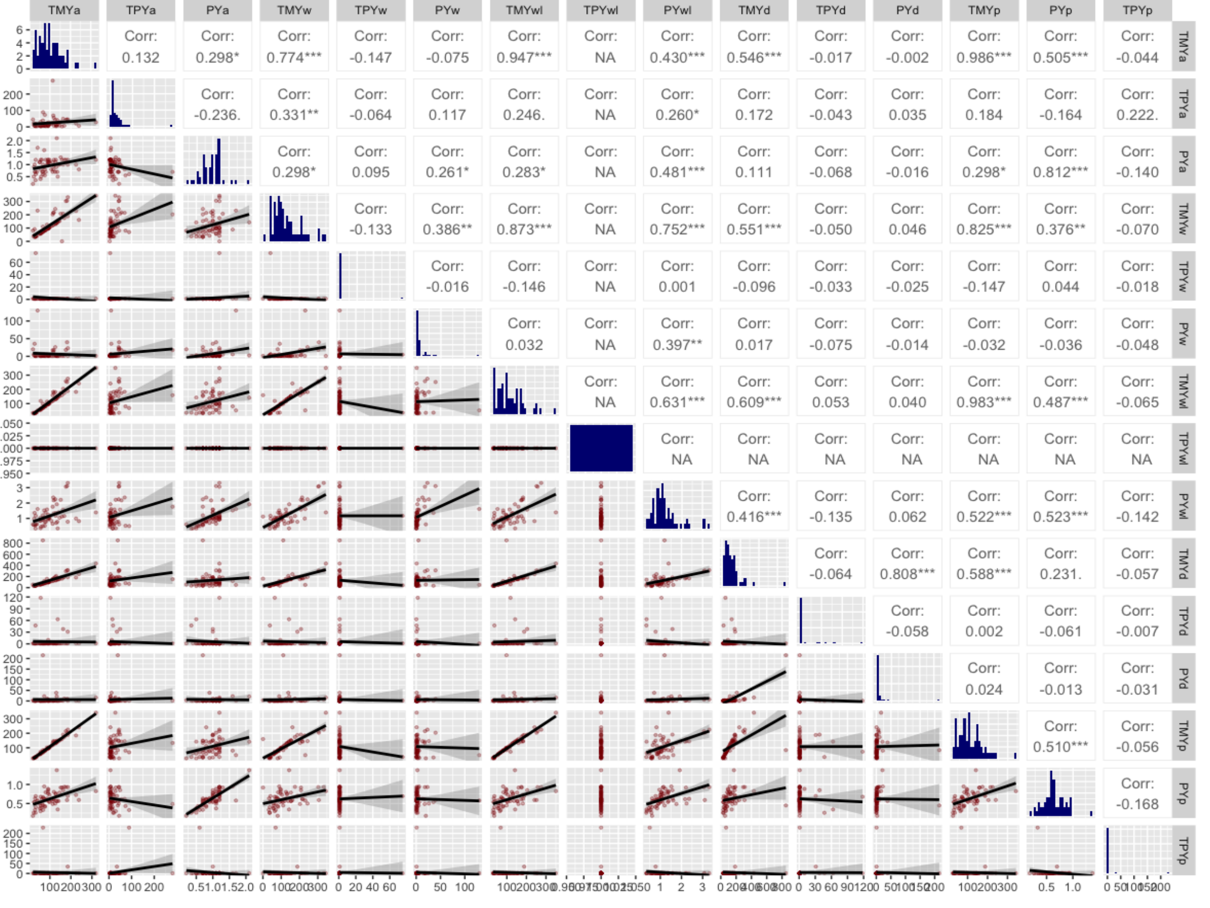

Figure 2. Correlogram of actual and estimated characteristics of atypical lactation curves.

Discussion

The common approach of the pattern of milk production in ruminants has been described as a curve with a first phase where milk yield increases to peak production followed by a decreasing phase denominated "typical curve" (Angeles-Hernandez *et al.*, 2014). However, an interesting phenomenon of study of the lactation curve is the existence of variations of this pattern production characterized by the absence of a peak of lactation with continuously decreasing yield since the beginning of lactation; this particular pattern of milk production has been called "atypical curve" (Angeles-Hernandez *et al.*, 2021).

In dairy sheep, the manifestation of atypical curves is more frequent, 30%- 50% of the lactation showed an atypical shape, which can be associated with the genetic improvement, nutritional management and level of intensification of dairy sheep farms (Cappio-Borlino *et al.*, 1997; Hernández *et al.*, 2017). Nevertheless, it must be considered that the lack of milk yield records in the first part of lactation can promote the presence of atypical curves due to the absence of an inflexion point in the decreasing phase of the curve (Angeles-Hernandez *et al.* 2021).

The fitting of the lactation curve provides useful information at farm level to evaluate the current performance and implement future nutritional, reproductive, genetic strategies based on the milk production pattern. The current study provides evidence that the shape of the lactation curve determines the performance of empirical and mechanistic models; showed all the fitted models difficulties to estimate the parameters to describe the lactation curve in atypical shapes. Which is in accordance with other author who referred that a typical lactation curves are not always easily fitted by conventional lactation models due to these were developed to describe a typical curve with ascending and descending phases (Macciotta *et al.*, 2005; Silvestre *et al.,* 2006).

In this sense, our results confirmed that empirical models can describe typical curves well. Also, they corroborate previous studies in which the WL model had a good performance in fitting lactations in sheep (Angeles-Hernandez *et al.*, 2013) and dairy cows (Olori *et al.*, 1999). On the other hand, the WD model struggled to estimate the TPY and PY for atypical curves. It can be associated with the WD model assuming that the maximum milk yield occurs at the onset of lactation between 1 and 2 days postpartum. However, based on the correlation values, the WD model is a good option for estimating TMY, which agrees with previous reports (Macciotta *et al.*, 2005).

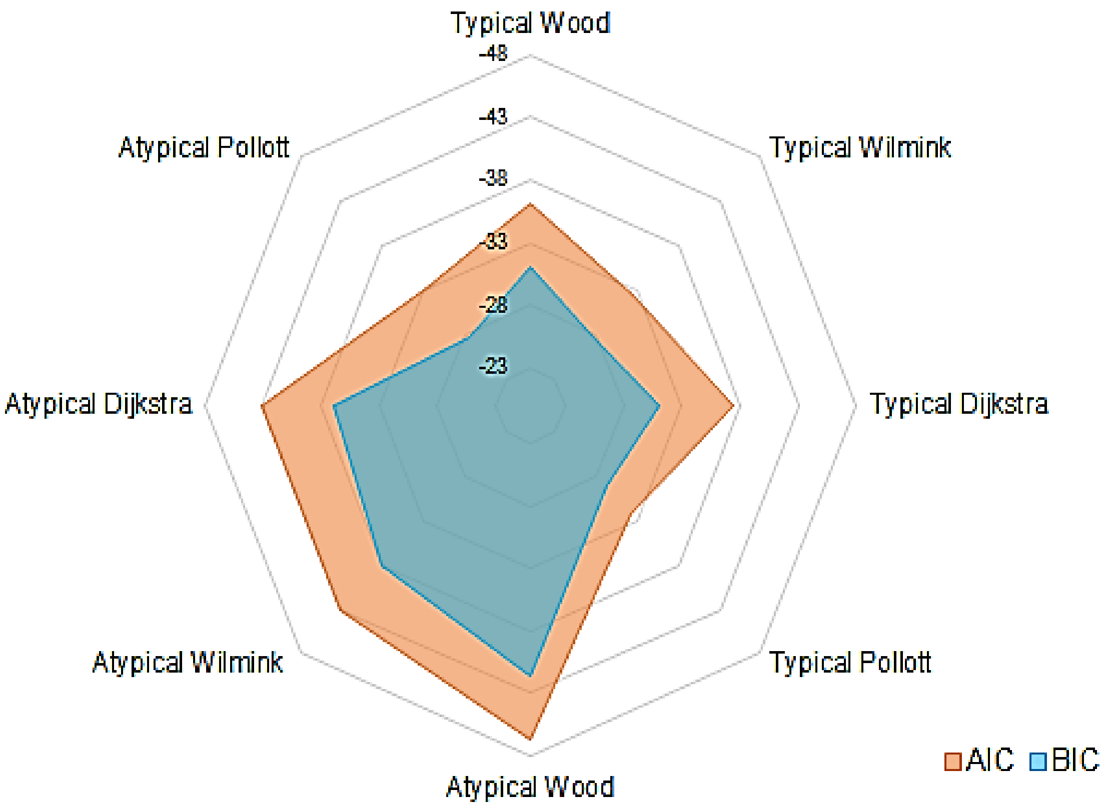

Figure 3. Goodness of fit evaluation according to AIC and BIC of typical and atypical dairy sheep lactation, using four models.

As mentioned above, the atypical curves are more frequent in sheep than in dairy cows. In the current work, the DJ showed difficulties in fitting atypical curve lactations, which can probably be associated with the fact that this model was developed to fit lactation curves with a pronounced peak of milk production. For this reason, the Djisktra model performed better than other models for typical sheep lactations.

Both mechanistic models showed some difficulties fitting atypical curves, which contrasts with Pollott and Goodwine (2000), who mentioned that mechanistic models have a greater power to adapt and can be used in a wide range of situations management. However, these deficiencies of the goodness of fit are compensated with the advantage of the biological interpretation of its parameters.

Conclusions

Fitting of the lactation curve is one of the most important applications of mathematical modeling in animal science and represents a fundamental step to understand the complex physiological process of lactation and improve decision making at farm level. Our results confirm that the shape of the lactation curve in dairy sheep affects the goodness of fit empirical and mechanistic models . In typical curves, the DJ model has the best performance and according to this, most of the criteria of goodness of fit. In contrast, the WD model best fits atypical curves according to MSPE, RMSPE, AIC, and BIC. Therefore, choosing the mathematical model to fit sheep lactation curves must consider the dairy herd characteristics that lead to atypical curve lactations.

Acknowledgment

The first author thanks to Conselho Nacional de Desenvolvimento Científico e tecnológico -CNPq for the grant awarded for the completion of doctoral studies. Also, the authors thanks to Coordenação de aperfeiçoamento de pessoal de nivel superior - CAPES for support the research stay of Lilian Paola Guevara Muñeton at the Instituto de Ciencias Agropecuarias of the Universidad Autónoma del Estado de Hidalgo, México

Conflict of interest: the authors declare that they have no conflict of interest

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